LIQUID FLOW AROUND A SELF-PROPELLED BODY

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Relatively little attention has been paid to the problem of flow around self-propelled bodies. This problem was discussed to a certain extent in [1], where an asymptotic expression was derived, according to which the velocity of two-dimensional flow in the wake of a self-propelled body moving at a constant velocity in a liquid which is at rest at infinity decreases with distance according to the $s^{-3/2}$ law, i.e., much faster than in the wake of an ordinary body experiencing the resistance offered by the liquid (in the latter case, the velocity decreases according to the $s^{-1/2}$ law [2, 3]).

It was assumed in [1] that the flow is symmetric with respect to the axis along which the body moves. In this case, the total force and the total moment of forces which the liquid exerts on the body are equal to zero, and it is not clear what quantity possessing a physical meaning could be related to the undetermined coefficient A in the asymptotic expression obtained in [1]. For this reason, it is impossible to answer on the basis of general considerations, for instance, the question of when this coefficient is not equal to zero (and, consequently, when the asymptotic expression for the flow velocity found in [1] does not become identically zero). It is even unclear whether A can be nonvanishing in any specific case of liquid flow around a self-propelled body.

Along with [1], one should also mention [2], where, with the aim of resolving the so-called Filon paradox, the first few terms of an asymptotic expansion of the stream function have been found for two-dimensional flow remote from an arbitrary cylindrical obstacle. Although the problem of liquid flow at large distances from a self-propelled body has not been raised in [2], one can obtain from the relationships found in [2] an asymptotic expression for the flow velocity which has the same form as the expression proposed in [1] by considering the case of symmetric flow and setting the resistance force equal to zero. However, the above-mentioned problem concerning the coefficient A remains unsolved also in this case.

We consider here the two-dimensional problem of stationary flow of a viscous incompressible liquid whose the velocity at infinity is $\mathbf{V}_\infty = (V_\infty, 0)$ around a cylinder with a mobile boundary (see Fig. 1). The flow is symmetric with respect to the x axis. The mobile boundary of the body acts as the motor. The problem of liquid motion throughout the entire flow region is solved in the approximation of small Reynolds numbers. In particular, we have derived an asymptotic expression for the flow velocity at large distances downstream from the body which has the same form as the expression proposed in [1].* The relationship between the coefficient A and the conditions at the body surface has been determined.

§1. Assume that a is the cylinder radius, ν is the kinematic viscosity coefficient, $\mathrm{Re} = aV_\infty/\nu$ is the Reynolds number, $x = X/a$, $y = Y/a$ are dimensionless Cartesian coordinates, $r = \sqrt{x^2 + y^2}$ is the dimensionless polar radius, θ is the polar angle, $f(\theta)$ is a certain odd function, defined over the interval $[-\pi, \pi]$; $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian operator, $\Psi = \psi a V_\infty$ is the stream function, $\mathbf{V} = V_\infty \mathbf{u}$ is the liquid flow velocity, $\Omega = \omega V_\infty/a$ is the vorticity, and $\mathbf{F}$ is the total force exerted by the liquid per unit length of the cylinder.

The dimensionless stream function of the flow under consideration constitutes the solution of the equation

$$\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\Delta\psi - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\Delta\psi = \frac{1}{\mathrm{Re}}\Delta^2\psi; \tag{1.1}$$

it satisfies the conditions

$$\psi = 0,\ \partial\psi/\partial r = -\varepsilon f \text{ for } r = 1; \tag{1.2}$$

$$\psi = 0,\ \partial^2\psi/\partial y^2 = 0 \text{ for } y = 0,\ |x| \geqslant 1; \tag{1.3}$$

$$\partial\psi/\partial x \to 0,\ \partial\psi/\partial y \to 1 \text{ as } r \to \infty. \tag{1.4}$$

* There is a printing error in this expression in [1].

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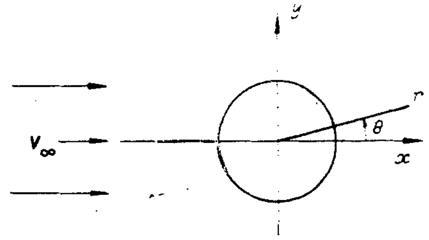

Fig. 1

Due to flow symmetry with respect to the x axis, the projection of the vector F on the y axis vanishes. Thus, $\mathbf{F} = (F, 0)$.

The function $f(\theta)$ can be assigned so that, for any Re, a value of ε can be found such that $F = 0$. This is confirmed by the following example. If

$$f(\theta) = -2\sin\theta,\ \varepsilon(\mathrm{Re}) = 1,$$

the solution of problem (1.1)-(1.4) is given by

$$\psi = (r - r^{-1})\sin\theta,$$

and the force acting per unit length of the cylinder, found from this solution, vanishes. There obviously exist other functions $f(\theta)$ such that $F = 0$ for a suitable choice of $\varepsilon(\mathrm{Re})$. It is assumed throughout this article that the points of the boundary move in such a manner that the condition $F = 0$ is satisfied. The total moment of forces exerted by the liquid per unit length of the cylinder is also equal to zero in the case under consideration.

The problem (1.1)-(1.4) is solved here in the approximation of small Reynolds numbers Re, using the method of asymptotic matching of internal and external expansions [4].

§2. Assume that Ψ, $\mathbf{u}$, ω and ε can be expanded in the following asmyptotic series for $\mathrm{Re} \to 0$:

$$\psi(r, \theta, \mathrm{Re}) \sim \psi_0(r, \theta) + g_1(\mathrm{Re})\psi_1(r, \theta) + \ldots; \quad (2.1a)$$

$$\mathbf{u}(r,\theta, \mathrm{Re}) \sim \mathbf{u}_0(r, \theta) + g_1(\mathrm{Re})\mathbf{u}_1(r, \theta) + \ldots; \quad (2.1b)$$

$$\omega(r, \theta, \mathrm{Re}) \sim \omega_0(r, \theta) + g_1(\mathrm{Re})\omega_1(r, \theta) + \ldots; \quad (2.1c)$$

$$\varepsilon(\mathrm{Re}) \sim \varepsilon_0 + g_1(\mathrm{Re})\varepsilon_1 + \ldots, \quad (2.2)$$

where $g_1(\mathrm{Re})$, $g_2(\mathrm{Re})$, ... is a certain sequence of functions such that $\lim_{\mathrm{Re}\to 0} g_1 = 0$, $\lim_{\mathrm{Re}\to 0} \frac{g_{m+1}}{g_m} = 0$, $m = 1, 2, \ldots$ The asymptotic expansions obtained for $\mathrm{Re} \to 0$ and fixed values of r and θ will be referred to as internal expansions. By substituting (2.1a) and (2.2) in (1.1)-(1.4) and retaining only the dominant terms, we define the zero-approximation problem:

$$\Delta^2\psi_0 = 0; \quad (2.3)$$

$$\psi_0 = 0,\ \partial\psi_0/\partial r = -\varepsilon_0 f \text{ for } r = 1; \quad (2.4)$$

$$\psi_0 = 0,\ \partial^2\psi_0/\partial y^2 = 0 \text{ for } y = 0,\ |x| \geqslant 1; \quad (2.5)$$

$$\partial\psi_0/\partial x \to 0,\ \partial\psi_0/\partial y \to 1 \text{ as } r \to \infty. \quad (2.6)$$

Assume that the function $f(\theta)$ can be expanded in a Fourier series:

$$f(\theta) = \sum_{m=1}^{\infty} f_m \sin m\theta. \quad (2.7)$$

By solving Eq. (2.3) and using (2.4) and (2.5), we find

$$\psi_0 = \left[a_0 r + \left(\frac{b_1}{4} - a_0\right) r^{-1} - \frac{b_1}{4} r^3 + (b_1 - \varepsilon_0 f_1 - 2a_0)\, r \ln r\right] \sin\theta + \sum_{m=2}^{\infty} \left\{\left(b_m - m c_m - \frac{\varepsilon_0}{2} f_m\right) r^{2-m} + \left[\frac{\varepsilon_0}{2} f_m + (m-1)\, c_m - \frac{m b_m}{m+1}\right] r^{-m} - \frac{b_m}{m+1} r^{2+m} + c_m r^m\right\} \sin m\theta, \quad (2.8)$$

where a_0, b_m, and c_m are arbitrary constants.

Having performed simple calculations, we satisfy ourselves that the total force per unit length of the cylinder exerted by the liquid in the approximation under consideration vanishes if, and only if, the following expression is satisfied:

$$b_1 - \varepsilon_0 f_1 - 2a_0 = 0.$$

The term proportional to $r \ln r \sin\theta$ vanishes in (2.8), as a result of which the well-known Stokes paradox [4], characteristic for the two-dimensional problem of stationary flow of a viscous liquid around a body for small Reynolds numbers, does not occur in our case.

By using (2.6), we finally find

$$\psi_0 = \frac{\varepsilon_0}{2} \sum_{m=1}^{\infty} f_m (r^{-m} - r^{2-m}) \sin m\theta, \tag{2.9}$$

where $\varepsilon_0 f_1/2 = -1$.

The scope of applicability of the obtained solution depends on the smallness of the inertial terms in comparison with the viscous terms in the liquid flow equations. After determining their values by means of (2.9), we can show that this condition is violated if $r\mathrm{Re} \sim 1$. Therefore, along with expansions (2.1), it is necessary to consider expansions which describe the flow in a certain external region while matching in some definite manner [4] the internal expansions. It should be noted that condition (2.6) at infinity, which is used above, coincides with the condition for matching the dominant term of the internal expansion of **u** with the dominant term of the external expansion of **u** which is determined below.

§3. Assume that Ψ, **u**, and ω can be expanded in the following asymptotic series as $\mathrm{Re} \to 0$:

$$\psi\left(\frac{\rho}{\mathrm{Re}}, \theta, \mathrm{Re}\right) \sim \frac{\rho}{\mathrm{Re}} \sin\theta + h_1(\mathrm{Re})\, \psi^{(1)}(\rho, \theta) + \ldots; \tag{3.1a}$$

$$\mathbf{u}\left(\frac{\rho}{\mathrm{Re}}, \theta, \mathrm{Re}\right) \sim \mathbf{i} + \mathrm{Re}\, h_1(\mathrm{Re})\, \mathbf{u}^{(1)}(\rho, \theta) + \ldots; \tag{3.1b}$$

$$\omega\left(\frac{\rho}{\mathrm{Re}}, \theta, \mathrm{Re}\right) \sim \mathrm{Re}^2 h_1(\mathrm{Re})\, \omega^{(1)}(\rho, \theta) + \ldots, \tag{3.1c}$$

where $\rho = \sqrt{\hat{x}^2 + \hat{y}^2}$, $\hat{x} = \mathrm{Re}\, x$, $\hat{y} = \mathrm{Re}\, y$; $\mathbf{i} = \mathbf{V}_\infty / V_\infty$; $h_1(\mathrm{Re})$, $h_2(\mathrm{Re})$,. . . is a certain sequence of functions such that

$$\lim_{\mathrm{Re}\to 0} \mathrm{Re}\, h_1 = 0, \quad \lim_{\mathrm{Re}\to 0} \frac{h_{m+1}}{h_m} = 0, \quad m = 1, 2, \ldots$$

The asymptotic expansions obtained for $\mathrm{Re} \to 0$ and fixed values of ρ and θ will be referred to as the external expansions.

As a result of flow symmetry with respect to the x axis, we have

$$\omega^{(m)} = 0 \quad \text{for} \quad \hat{y} = 0, \ \hat{x} \neq 0, \ m = 1, 2, \ldots; \tag{3.2}$$

$$\psi^{(m)} = 0 \quad \text{for} \quad \hat{y} = 0, \ \hat{x} \neq 0, \ m = 1, 2, \ldots \tag{3.3}$$

For $\rho \to \infty$, $\mathbf{u} \to \mathbf{i}$, so that the following must hold:

$$\omega^{(m)} \to 0 \quad \text{as} \quad \rho \to \infty, \ m = 1, 2, \ldots; \tag{3.4}$$

$$u_x^{(m)} \to 0, \ u_y^{(m)} \to 0 \quad \text{as} \quad \rho \to \infty, m = 1, 2, \ldots, \tag{3.5}$$

where $u_x^{(m)}$ and $u_y^{(m)}$ are the x and y components of the vector $\mathbf{u}^{(m)}$, respectively.

We rewrite Eq. (1.1) in terms of the variables $\hat{x}$ and $\hat{y}$ and substitute in it (3.1a); retaining the dominant terms, we then obtain

$$\partial\omega^{(1)}/\partial\hat{x} = \hat{\Delta}\omega^{(1)},$$

where $\hat{\Delta} = \partial^2/\partial\hat{x}^2 + \partial^2/\partial\hat{y}^2$. The solution of this equation that satisfies conditions (3.2) and (3.4) is given by

$$\omega^{(1)} = e^{\hat{x}/2} \sum_{m=1}^{\infty} A_m K_m(\rho/2) \sin m\theta, \tag{3.6}$$

where K_m is MacDonald's function, and A_m are arbitrary constants.

§4. We now pass to the problem of matching the internal and the external expansions. Assume that Q is a set of functions q(r, θ, Re), which, for Re→0, can be expanded in the following asymptotic series:

$$q \sim \alpha_0(\mathrm{Re})q_0(r,\ \theta) + \alpha_1(\mathrm{Re})q_1(r,\ \theta) + \ldots,$$
$$q \sim \beta_0(\mathrm{Re})q^{(0)}(\rho,\ \theta) + \beta_1(\mathrm{Re})q^{(1)}(\rho,\ \theta) + \ldots,$$

where $\alpha_0(\mathrm{Re})$, $\alpha_1(\mathrm{Re})$, . . ., $\beta_0(\mathrm{Re})$, $\beta_1(\mathrm{Re})$, . . . are certain sequences of functions, such that

$$\lim_{\mathrm{Re}\to 0}\frac{\alpha_{m+1}}{\alpha_m} = 0,\ \lim_{\mathrm{Re}\to 0}\frac{\beta_{m+1}}{\beta_m} = 0,\ m = 0,1,2,\ldots$$

Let us define the operators I_{α_m} and E_{β_n}:over the set Q:

$$I_{\alpha_0}q = \alpha_0\lim_{\mathrm{Re}\to 0}\frac{q}{\alpha_0} \quad \text{for fixed values of r and } \theta$$

$$I_{\alpha_m}q = I_{\alpha_{m-1}}q + \alpha_m\lim_{\mathrm{Re}\to 0}\frac{q - I_{\alpha_{m-1}}q}{\alpha_m} \quad \text{for fixed values of r and } \theta$$
$$m = 1,\ 2\ldots;$$

$$E_{\beta_0}q = \beta_0\lim_{\mathrm{Re}\to 0}\frac{q}{\beta_0} \quad \text{for fixed values of } \rho \text{ and } \theta$$

$$E_{\beta_n}q = E_{\beta_{n-1}}q + \beta_n\lim_{\mathrm{Re}\to 0}\frac{q - E_{\beta_{n-1}}q}{\beta_n} \quad \text{for fixed values of } \rho \text{ and } \theta$$
$$n = 1,\ 2,\ldots$$

The principle of asymptotic joining [4] is used below for matching the internal and external expansions. According to this principle, the following relationship holds for the operators I_{α_m} and E_{β_n}:

$$I_{\alpha_m}E_{\beta_n}q = E_{\beta_n}I_{\alpha_m}q. \tag{4.1}$$

In practice, the matching involves a choice of internal and external variables, determination of the comparison functions, etc. [4]. As a rule, any specific solution of these problems has the character of an assumption, which is justified by a successful matching.

In the case under consideration, the dominant terms of the internal and external expansions of ω are matched if $h_1(\mathrm{Re}) = \mathrm{Re}^{n-2}$, where n = 2, 3, 4, . . . is the number of the first nonvanishing coefficient f_n in (2.7) (not counting f_1). Assume that $f_2 \neq 0$ and, correspondingly, $h_1(\mathrm{Re}) = 1$. Using (2.9), we find

$$E_{\mathrm{Re}^2}I_1\omega = -E_{\mathrm{Re}^2}\Delta\psi_0 = -2\varepsilon_0 f_2\mathrm{Re}^2\rho^{-2}\sin 2\theta.$$

Considering that $E_{\mathrm{Re}^2}I_1\omega = I_1E_{\mathrm{Re}^2}\omega$ and using (3.6), we obtain

$$A_2 = -\varepsilon_0 f_2/4,\ A_k = 0,\ k = 3,\ 4,\ \ldots$$

Thus,

$$\omega^{(1)} = e^{\frac{1}{2}\rho\cos\theta}\left\{A_1K_1(\rho/2) - \frac{1}{2}\varepsilon_0 f_2K_2(\rho/2)\cos\theta\right\}\sin\theta. \tag{4.2}$$

Now consider the Navier–Stokes equation and the equation of continuity:

$$(\mathbf{u}\cdot\hat{\nabla})\mathbf{u} = -\hat{\nabla}p + \hat{\Delta}\mathbf{u}; \tag{4.3}$$
$$\hat{\nabla}\cdot\mathbf{u} = 0, \tag{4.4}$$

where $\hat{\nabla} = (\partial/\partial\hat{x},\ \partial/\partial\hat{y})$; $p\sigma V_\infty^2$ is the pressure, and σ is the density of the liquid.

Let us expand p in the following asymptotic series for Re→0

$$p \sim p_\infty + \mathrm{Re}p^{(1)}(\rho,\ \theta) + \mathrm{Re}h_2(\mathrm{Re})p^{(2)}(\rho,\ \theta) + \ldots, \tag{4.5}$$

where p_∞ is the value of p at infinity.

By substituting (3.1b) and (4.5) in (4.3) and (4.4) and retaining only the dominant terms, we obtain

$$\partial\mathbf{u}^{(1)}/\partial\hat{x} = -\hat{\nabla}p^{(1)} + \hat{\Delta}\mathbf{u}^{(1)}; \tag{4.6}$$

$$\hat{\nabla}\cdot \mathbf{u}^{(1)} = 0. \tag{4.7}$$

Equations (4.6) and (4.7), as is known [5], have the following solutions:

$$u_x^{(1)} = \partial\Phi/\partial\hat{x} + \partial\chi/\partial\hat{x} - \chi; \tag{4.8}$$

$$u_y^{(1)} = \partial\Phi/\partial\hat{y} + \partial\chi/\partial\hat{y}; \tag{4.9}$$

$$p^{(1)} = -\partial\Phi/\partial\hat{x}, \tag{4.10}$$

if

$$\hat{\Delta}\Phi = 0; \tag{4.11}$$

$$\partial\chi/\partial\hat{x} = \hat{\Delta}\chi. \tag{4.12}$$

By solving Eqs. (4.11) and (4.12), considering that $\partial\chi/\partial\hat{y} = \omega^{(1)}$ and using (3.5), (4.2), and (4.8)-(4.10), we obtain

$$u_x^{(1)} = a'\rho^{-1}\cos\theta - \sum_{n=2}^{\infty}(b_n'\cos n\theta + c_n'\sin n\theta)\rho^{-n} + \tag{4.13}$$

$$+ e^{\frac{1}{2}\rho\cos\theta}\left\{\left(A_1 - \frac{1}{4}\varepsilon_0 f_2\right)K_0(\rho/2) + \left(A_1 - \frac{1}{2}\varepsilon_0 f_2\right)K_1(\rho/2)\cos\theta - \frac{1}{4}\varepsilon_0 f_2 K_2(\rho/2)\cos 2\theta\right\},$$

$$u_y^{(1)} = a'\rho^{-1}\sin\theta + \sum_{n=2}^{\infty}(c_n'\cos n\theta - b_n'\sin n\theta)\rho^{-n} +$$

$$+ e^{\frac{1}{2}\rho\cos\theta}\left\{A_1 K_1(\rho/2) - \frac{1}{2}\varepsilon_0 f_2 K_2(\rho/2)\cos\theta\right\}\sin\theta;$$

$$p^{(1)} = -a'\rho^{-1}\cos\theta + \sum_{n=2}^{\infty}(b_n'\cos n\theta + c_n'\sin n\theta)\rho^{-n}, \tag{4.14}$$

where a', b'_n, and c'_n are arbitrary constants.

By using (2.9), we find

$$E_{\mathrm{Re}}I_1\frac{\partial\psi}{\partial x} = \frac{1}{2}\varepsilon_0 f_2 \mathrm{Re}\rho^{-1}(\sin 3\theta - \sin\theta).$$

In correspondence with (4.1), $E_{\mathrm{Re}}I_1\,(\partial\Psi/\partial x) = I_1E_{\mathrm{Re}}(\partial\Psi/\partial x)$, whence

$$a' = \varepsilon_0 f_2 - 2A_1,\ b_2' = -2\varepsilon_0 f_2,$$
$$b_m' = 0,\ m = 3, 4, \ldots,\ c_n' = 0,\ n = 2, 3, \ldots$$

§5. It can be shown [1] that

$$F = \mu V_\infty\left\{\frac{1}{\mathrm{Re}}\oint\omega d\hat{x} - \oint(p - p_\infty + u_x - 1)d\hat{y} + \oint u_y(u_x - 1)d\hat{x} - \oint(u_x - 1)^2 d\hat{y}\right\},$$

where $\mu = \sigma\nu$ is the viscosity of the liquid, and u_x and u_y are the x and y components of the vector **u**, respectively.

The integration is performed in the flow phane along a certain contour S containing the cross section of the cylinder. We shall use a circle with the radius L whose center is at the coordinate origin as the contour S. Using, instead of u_x, u_y, ω, and p, their external expansions, we find as $\mathrm{Re} \to 0$

$$F \sim \mu V_\infty \mathrm{Re}\left[\oint\omega^{(1)}d\hat{x} - \oint(p^{(1)} + u_x^{(1)})d\hat{y}\right] + \ldots \text{ for } \mathrm{Re}\to 0. \tag{5.1}$$

Since $F = 0$, all the terms of expansion (5.1) also must vanish. By using (4.2), (4.13), and (4.14), we obtain

$$\oint\omega^{(1)}d\hat{x}\to 0,\ \oint(p^{(1)} + u_x^{(1)})d\hat{y}\to 2\pi(2A_1 - \varepsilon_0 f_2) \text{ for } L\to\infty.$$

Thus, as a result of the fact that the dominant term of expansion (5.1) vanishes, we have

$$A_1 = \varepsilon_0 f_2/2.$$

§6. We can now write the final expressions for $u_x^{(1)}$, $u_y^{(1)}$, and $\omega^{(1)}$

$$u_x^{(1)} = 2\varepsilon_0 f_2 \rho^{-2} \cos 2\theta + \frac{1}{4} \varepsilon_0 f_2 e^{\frac{1}{2}\rho\cos\theta} [K_0(\rho/2) - K_2(\rho/2)\cos 2\theta]; \quad (6.1)$$

$$u_y^{(1)} = 2\varepsilon_0 f_2 \rho^{-2} \sin 2\theta + \frac{1}{2} \varepsilon_0 f_2 e^{\frac{1}{2}\rho\cos\theta} [K_1(\rho/2) - K_2(\rho/2)\cos\theta]\sin\theta; \quad (6.2)$$

$$\omega^{(1)} = \frac{1}{2} \varepsilon_0 f_2 e^{\frac{1}{2}\rho\cos\theta} [K_1(\rho/2) - K_2(\rho/2)\cos\theta]\sin\theta. \quad (6.3)$$

Considering that $\partial\Psi^{(1)}/\partial\hat{x} = -u_y^{(1)}$, and $\partial\Psi^{(1)}/\partial\hat{y} = u_x^{(1)}$ and using (3.3), (6.1), and (6.2), we find

$$\psi^{(1)} = 2\varepsilon_0 f_2 \rho^{-1} \sin\theta - \varepsilon_0 f_2 e^{\frac{1}{2}\rho\cos\theta} K_1(\rho/2)\sin\theta. \quad (6.4)$$

Using (2.9) and (6.1)-(6.4), we obtain expressions Ψ, u_x, u_y, and ω for which are equally suitable for the entire flow region. Using the method of additive composition [4], we find

$$\psi \approx (I_1 + E_1 - I_1E_1)\,\psi = \psi_0 + \psi^{(1)} + \frac{1}{2}\varepsilon_0 f_2 \sin 2\theta;$$

$$u_x \approx (I_1 + E_{\mathrm{Re}} - I_1E_{\mathrm{Re}})\,u_x = u_{0x} + \mathrm{Re}\,u_x^{(1)} + \frac{1}{2}\varepsilon_0 f_2 r^{-1}(\cos\theta + \cos 3\theta); \quad (6.5)$$

$$u_y \approx (I_1 + E_{\mathrm{Re}} - I_1E_{\mathrm{Re}})\,u_y = u_{0y} + \mathrm{Re}\,u_y^{(1)} - \frac{1}{2}\varepsilon_0 f_2 r^{-1}(\sin\theta - \sin 3\theta); \quad (6.6)$$

$$\omega \approx (I_1 + E_{\mathrm{Re}^2} - I_1E_{\mathrm{Re}^2})\,\omega = \omega_0 + \mathrm{Re}^2\omega^{(1)} + 2\varepsilon_0 f_2 r^{-2}\sin 2\theta, \quad (6.7)$$

where u_{0x} and u_{0y} are the x and y components of the vector $\mathbf{u}_0$, respectively.

The relationships derived above make it possible, in particular, to answer the question concerning the asymptotic behavior of the flow velocity and vorticity at large distances downstream from the body under consideration (for small Re values). By using (6.5)-(6.7) and passing to dimensional quantities, we obtain the following asymptotic expressions:

$$\mathbf{V} - \mathbf{V}_\infty \sim \mathbf{i}\frac{A}{X^{3/2}}\left(\frac{V_\infty Y^2}{\nu X} - 2\right) e^{-\frac{V_\infty Y^2}{4\nu X}},$$

$$\Omega \sim B\frac{Y}{X^{5/2}}\left(\frac{V_\infty Y^2}{\nu X} - 6\right) e^{-\frac{V_\infty Y^2}{4\nu X}}$$

for $X \to \infty$ and a fixed value of Y^2/X. Here $A = \frac{1}{2}\pi^{1/2}\varepsilon_0 f_2 a \nu^{1/2} V_\infty^{1/2}$; $B = \frac{1}{4}\pi^{1/2}\varepsilon_0 f_2 a \nu^{-1/2} V_\infty^{3/2}$.

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